Synthetic Aperture Radar Interferometry Using Ground Slope Vector to Phase Unwrapping.

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1. Abstract

This paper deals with 2-dimentional synthetic aperture radar interferometry. Because of high sensitivity to noise, we propose a two-step phase unwrapping approach. The purpose of preprocessing is to organize the phase by an adaptive filter to improve the estimation of the ground slope vector. The main process smoothes the phase, using the ground slope components, and unwraps the signal. We present numerical and experimental results for synthetic data.

2. Introduction

Synthetic aperture radar (SAR) interferometry is a powerful technique allowing the generation of digital ground models (DGM). To get a high quality DGM, we must have a low-noise interferometric image. To unwrap the phase, it is thus necessary combine two single complex views, obtained from different observation angles, of the same scene. In a low-noise case the unwrapping would be easy; however, local errors due to noise result in global errors due to causal nature of unwrapping. Several methods exist [3] [4] to avoid the propagation of local errors; for example [2], detects fringe lines with edge enhancing and then adds 2π whenever the path crosses a fringe line.
Our approach consists in a preprocessing which is a simple partial unwrapping method, followed by the main algorithm, which unwraps and smoothes the signal by working on the ground slope vector components.

3. Statement of the problem

The problem can be defined as follows :

$$\boxed{\psi = \theta + 2k\pi + b} \quad \textbf{(1)}$$

where ψ represents the unwrapped phase, θ the wrapped noisy phase, k an integer for unwrapping, and b the noise.

To restore the signal we must find the best k (phase unwrapping) and then filter the resulting ψ to improve the phase estimate. We then find the DGM.

The solution we propose is based on signal slopes. Indeed, we make the strong assumption that the ground slopes are very smooth, except ground irregularity. Our unwrapping algorithm works in two steps :

- ✓ The first step (preprocessing) is used to get a better estimation of the gradient of the signal i.e. the ground slopes. We obtain a partially unwrapped phase. This algorithm directly works on observed signal θ .
- ✓ The second step (main processing), the original contribution of this paper, is to use the ground slope vector components to unwrap and smooth the phase.

4. Main Processing

The main idea is to smooth the ground slopes while preserving edges [5] due to wrapping and some ground irregularities. We use the assumption that the ground slopes are very smooth.
Under the assumption that ground is modeled as a non-separable function, we mathematically define :

$$\boxed{\begin{array}{l} Z = \begin{pmatrix} Z_x \\ Z_y \end{pmatrix} = \begin{pmatrix} D_x\psi \\ D_y\psi \end{pmatrix} \\ \psi = M_x Z_x + cste \\ or \\ \psi = M_y Z_y + cste \end{array}} \quad \textbf{(2)}$$

with Z_x and Z_y the components of the ground slope vector, or first order derivatives of ψ , and M_x the integration matrix in the *x* axis direction (i.e. M_y in the *y* axis direction).

M_x is a bloc-matrix :

$$Mb_x = \begin{pmatrix} 1 & 0 & & 0 \\ \bullet & \bullet & & \\ \bullet & & \bullet & 0 \\ 1 & \bullet & \bullet & 1 \end{pmatrix} \qquad M_x = \begin{pmatrix} Mb_x & 0 & & 0 \\ 0 & \bullet & & \\ & & \bullet & 0 \\ 0 & & 0 & Mb_x \end{pmatrix} \tag{3}$$

To estimate ψ we need to estimate one of the slope vector components Z_x or Z_y. Yet, we need to estimate both of the Z_x and Z_y components in order to avoid artifacts due to integration, which are lines with the same orientation as the integration direction.

To estimate the Z_x and Z_y components of Z from θ and k while taking into account the assumption of homogeneous ground slopes, we minimize the criterion of the form :

$$\boxed{J(Z_x) = \left\| M_x Z_x - \theta - 2k_x\pi \right\|^2 + \lambda \sum_{Z_x} \varphi\left(\frac{1}{\delta}\left|\nabla Z_x\right|\right) + \mu \sum_{Z_X} \left(D_x Z_y - D_y Z_x\right)^2} \tag{4}$$

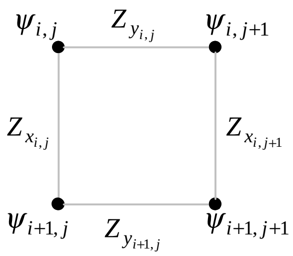
$$\boxed{J(Z_y) = \left\| M_y Z_y - \theta - 2k_y\pi \right\|^2 + \lambda \sum_{Z_y} \varphi\left(\frac{1}{\delta}\left|\nabla Z_y\right|\right) + \mu \sum_{Z_y} \left(D_x Z_y - D_y Z_x\right)^2} \tag{5}$$

where $\left|\nabla Z_x\right|$ is the modulus of the gradient component Z_x, i.e. the second order derivative of ψ, λ and μ represent the regularization parameters that control the relative weighting between the data term and the regularization terms. The potential function φ, that is applied on the gradient of the slopes, is chosen so as to preserve edges [5] due to wrapping and to ground irregularity. The obtained sharp discontinuities, due to wrapping, are easily removed by a non linear filter like a median. Consequently, we introduce the parameter δ as a threshold level from which we decide to preserve or smooth the discontinuities. To preserve discontinuities, the φ functions must satisfy some properties [1]. The third term of the criterion :

$\mu \sum_{Z_y} \left(D_y Z_x - D_x Z_y\right)^2$ represents a constraint on the second partial derivative of ψ with respect to *x* and *y* :

$D_y Z_x = D_y D_x \psi$, i.e. $D_x Z_y = D_x D_y \psi$.

This constraint assumes that whatever the way to calculate a pixel from an other one, the solution must be equal :



To calculate $\psi_{i+1,j+1}$ from $\psi_{i,j}$:

$$\psi_{i+1,j+1} = \psi_{i,j} + Z_{y_{i,j}} + Z_{x_{i,j+1}}$$

and :

$$\psi_{i+1,j+1} = \psi_{i,j} + Z_{x_{i,j}} + Z_{y_{i+1,j}}$$

therefore :

$$Z_{y_{i,j}} + Z_{x_{i,j+1}} - Z_{x_{i,j}} - Z_{y_{i+1,j}} = 0$$
$$Z_{x_{i,j+1}} - Z_{x_{i,j}} = Z_{y_{i+1,j}} - Z_{y_{i,j}}$$
$$D_y Z_x = D_x Z_y$$

eventually :

$$\boxed{D_y D_x \psi = D_x D_y \psi}$$

If this equality is verified then the artifacts, due to integration, are avoided.

We use a derivative method to minimize the criterion (4) and (5).

Consequently, the minimum of $J(Z_x)$, if it exists, satisfies the following equation :

$$\boxed{\frac{\partial J(Z_x)}{\partial Z_x} = 0 \Leftrightarrow \left(M_x^T M_x - \tfrac{\lambda}{\delta^2}\Delta_{pond} + \mu D_y^T D_y\right) Z_x = M_x^T(\theta + 2k\pi) + \mu D_y^T D_x Z_y} \tag{6}$$

and the minimum of $J(Z_y)$, if its exists, satisfies the following equation :

$$\boxed{\frac{\partial J(Z_y)}{\partial Z_y} = 0 \Leftrightarrow \left(M_y^T M_y - \tfrac{\lambda}{\delta^2}\Delta_{pond} + \mu D_x^T D_x\right) Z_y = M_y^T(\theta + 2k\pi) + \mu D_x^T D_y Z_x} \tag{7}$$

With Δ_{pond} as a discrete approximation of weighted laplacian [1], weighted by the coefficients :

$$\boxed{d_{x_{i,j}}(Z_x) = \frac{\varphi'\left(\frac{1}{\delta}\left|\nabla Z_x\right|_{(i,j)}\right)}{\frac{2}{\delta}\left|\nabla Z_x\right|_{(i,j)}}} \qquad \textbf{(8)}$$

The variable d is then a map of the edge locations, i.e. for $d_{y_{i,j}}(Z_y)$.

The algorithm is :

θ = Preprocessing(θ), see § 5.

$\psi^0 = 0, d^0(Z_x) = 1$

Repeat

- Z_x^{n+1}, solving (6) with a conjugate gradient algorithm.
- Z_y^{n+1}, solving (7) with a conjugate gradient algorithm.
- $Z_{med}^{n+1}\big|_x = median\left(Z_x^{n+1}\right)$
- $Z_{med}^{n+1}\big|_y = median\left(Z_y^{n+1}\right)$
- $\psi^{n+1} = M_x Z_{med}^{n+1}\big|_x$
- $k^{n+1} = \mathrm{nint}\left(\frac{1}{2\pi}\left(\psi^{n+1} - \theta\right)\right)$
- $d^{n+1}(Z_x)$ *and* $d^{n+1}(Z_y)$, calculating (8).

Until convergence.

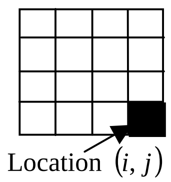
With nint as the nearest integer function.

Notice that $Z_{med}^{n+1}\big|_x$ is obtained by a median filtering of Z_x^{n+1}. Moreover, the result is not very sensitive to the size of the median filter.

The presence of noise can result in many wrapping in a small range, and we cannot estimate slopes in the areas where jumps of 2π are too close together. For this reason, we introduce a preprocessing stage to reorganize the data.

5. Preprocessing

The basic idea is to organize the data such that there is no difference between a pixel and its previous neighbors bigger than π. Because of the causal aspect of unwrapping, we use the following causal neighborhood of the scanned pixel :



□ Neighborhood of a pixel

For a neighborhood of size t, we estimate the local value of k :

$$\boxed{k(\theta_i, t) = \mathrm{nint}\left(\frac{\mathrm{E}(t) - \theta_i}{2\pi}\right)} \qquad \textbf{(9)}$$

where nint is the nearest integer and $\mathrm{E}(t)$ is the mean of neighborhood pixels. Notice that k depends on neighborhood size t. Moreover, the result is sensitive to the noise level and consequently to t.

The solution we take is to adapt t during the preprocessing.

6. Experimental results

Experiments on our algorithm were conducted for synthetic data, a Gaussian signal (Fig 1), and for real data, ERS1 interferogram of ETNA volcano's crater (Fig 7, 8).

For the original Gaussian signal, the uniform noise was high enough to force the preprocessing algorithm into traps (Fig 3). But the main preprocessing stage, working with the ground slopes (Fig 4), unwraps and smoothes the phase efficiently (Fig 4).

7. Acknowledgements

Many thanks to D. Massonnet of CNES for providing the ERS1 interferogram of ETNA volcano 's crater.

8. Conclusion

In this paper, we propose a new SAR interferometry algorithm for phase that uses smoothed ground slope components. The experimental results on 2-D signals are encouraging. We also seek to improve our method through ground irregularity preservation and a better homogeneous slope model.

9. References

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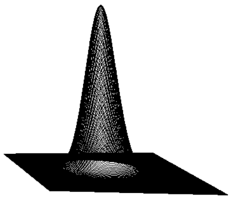

Fig. 1 : Original gaussian.

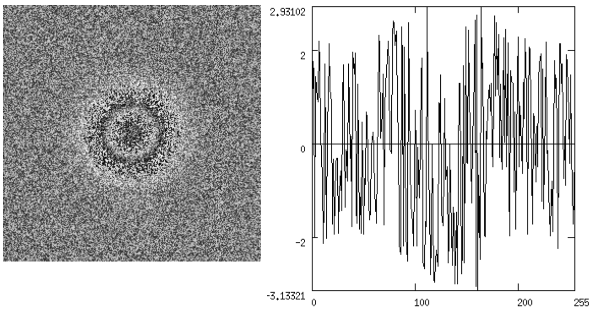

Fig. 2 : Wrapped noisy gaussian and a cross-section.

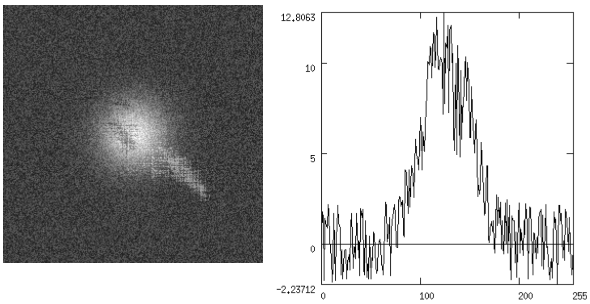

Fig. 3 : Preprocessing : partially unwrapped phase in order to get a better estimation of the slope components.

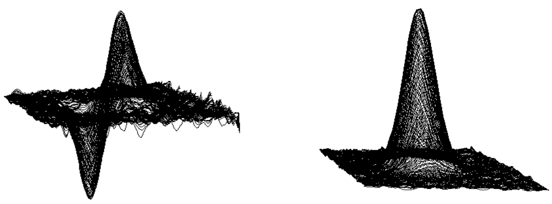

Fig. 4 : Ground slope component Z_x and Unwrapped signal.

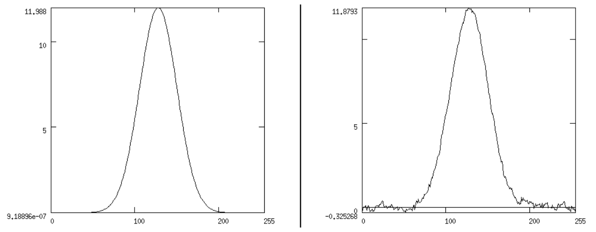

Fig. 6 : Cross sections of the original gaussian (left side) and of the Unwrapped and smoothed signal (right side).

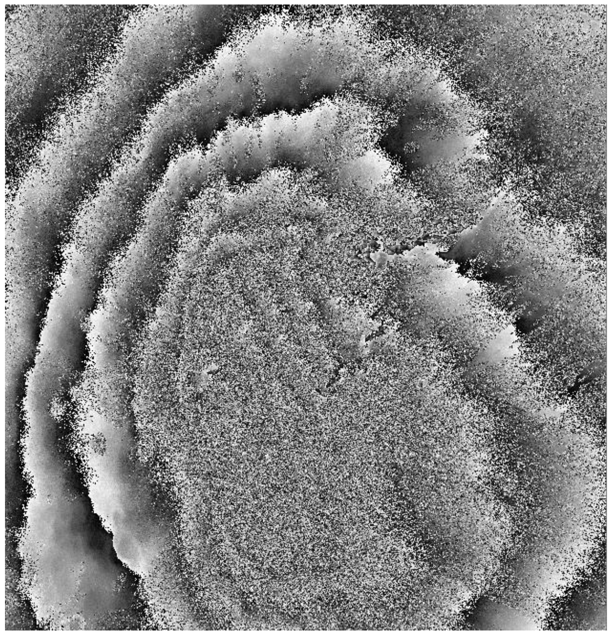

Fig. 7 : The ERS1 interferogram of ETNA volcano 's crater.

Fig. 8 : The unwrapped ETNA volcano 's crater.